The Adjustable Balance Mortgage: Reducing the Value of the Put*

Brent W. Ambrose, Ph.D.
Institute for Real Estate Studies
Smeal College of Business
The Pennsylvania State University
University Park, PA 16802
(814) 867-0066
bwa10@psu.edu

and

Richard J. Buttimer, Jr., Ph.D.
Department of Finance
Belk College of Business Administration
The University of North Carolina at Charlotte
9201 University City Boulevard
Charlotte, NC 28223-0001
(704) 687-7695
buttimer@uncc.edu

September 11, 2009

The Adjustable Balance Mortgage:
Reducing the Value of the Put

ABSTRACT

We propose a new mortgage contract that endogenously captures the risk of house price declines to minimize default risk resulting from changes in the underlying asset value while still retaining contract rates near the cost of a standard fixed-rate mortgage. By reducing the role of the legal system in mitigating house price risk, the new mortgage reduces the negative externalities and social costs arising from defaults. In other words, the new mortgage minimizes the need to use the legal foreclosure system to deal with the economic risk of house price declines. In addition, we utilize the model to analyze the potential success of the mortgage modification proposals that are designed to deal with the current foreclosure crisis. Consistent with the high redefault rate experienced by many modified loans, our numerical analysis shows that modification programs with one-time principal adjustments do not remove the potential for future financially motivated default.

1 Introduction

Why do borrowers default? Modern option pricing theory shows that the preponderance of defaults are a result of the home price declining to less than the value of the outstanding mortgage. Prior to the development of this theory, however, mortgage default was assumed to result from either a moral failing on the part of the borrower or from cash flow problems that prevented the borrower from repaying the debt. As neither cause could be hedged, the mortgage contract developed out of the legal traditions of contract enforcement in order to minimize borrower default risk. In the U.S., these legal traditions center on the use of foreclosure laws to take the underlying property from a borrower in default in order to satisfy the lender's loss on the defaulted debt.

Periods of economic turmoil, however, illustrate with stark clarity the inherent inefficiencies of financial contracts designed for previous eras and provide the impetus for financial innovation and reform. For example, prior to the Great Depression, the typical U.S. mortgage was a 5-year, interest-only note. The non-amortizing feature of these loans exposed lenders to significant default risk, which was confirmed when housing prices plummeted following the stock market crash of 1929, leading to massive mortgage foreclosures during the 1930s.[1] As part of the New Deal legislation enacted to respond to the financial crisis of the Great Depression, the new Federal Housing Administration (FHA) led the efforts to create the 30-year, fully amortizing fixed-rate mortgage (FRM) that became the standard instrument for the next 75-years.[2]

The current mortgage default crisis resulting from the housing bubble of 2004 to 2006 demonstrates the dramatic deadweight costs associated with using the legal system (i.e. foreclosure) to minimize borrower defaults, and suggests the need for a new mortgage that will eliminate or significantly reduce the risks associated with volatile property markets. The financial crisis also demonstrates a fundamental flaw in the current housing finance system: the threat of foreclosure is not sufficient to prevent widespread default when house prices fall significantly. Thus, we propose a new "adjustable balance" mortgage contract that endogenously captures the risk of house price declines to minimize default risk resulting from changes in the underlying asset value while still

---

[1] See Clauretie and Sirmans (2006) for an excellent brief history of the U.S. mortgage market. To reinforce the risk associated with the pre-1930's mortgages, Clauretie and Sirmans (2006) report that Savings and Loan associations had foreclosed on one-fifth of their mortgage loans by 1935.

[2] In another example of mortgage innovation arising during a financial crisis, the now common adjustable-rate mortgage (ARM) became widely accepted during the high inflationary period of the late 1970s and early 1980s as it allowed lenders to mitigate the significant interest rate risk associated with the traditional fixed-rate mortgage.

retaining contract rates near the cost of a standard mortgage. By reducing the role of the legal system, the new mortgage reduces the negative externalities and social costs arising from defaults resulting from house price risk. In other words, the new mortgage minimizes the need to use the legal system to deal with the economic risk of house price declines.

Our analysis proceeds as follows: In Section 2, we describe the new Adjustable Balance Mortgage (ABM) concept. Section 3 presents a formal model for pricing this mortgage. Our pricing model allows for fully endogenous borrower prepayment and default, and thus allows us to solve for the equilibrium contract interest rate across products. Thus, we are able to explicitly price the automatic modification features of this product relative to the traditional fixed-rate mortgage (FRM) contract. Section 4 presents the numerical analysis of the ABM and FRM contracts. Section 5 follows with a discussion of the impact of incorporating default transaction costs into the analysis. In Section 6, we utilize the insights derived from the numerical analysis to comment on the current mortgage modification policies being used to combat the mortgage default crisis. Finally, Section 7 concludes.

2 The Adjustable Balance Mortgage (ABM)

We begin with the notion that all mortgages have two primary sources of uncertainty: interest rates and house prices. The contractual treatment of these risks, however, is asymmetric. The markets have, as a whole, directly addressed the issue of whom bears interest rate risk by developing a menu of mortgage contracts. For example, borrowers can elect to take interest rate risk through the adjustable-rate mortgage (ARM), or select a fixed-rate mortgage (FRM) and pay the lender (through higher contract rates) to take the interest rate risk. Lenders can elect to issue FRMs or ARMs depending upon their risk preferences. Competitive forces in the lending market have resulted in a wide variety of mortgage alternatives (with different adjusting periods, interest rate caps and floors, and payment options) that allow lenders and borrowers to contract on the degree of interest rate risk each side wishes to bear. The market, however, has not evolved similar flexibility with respect to housing price risk. Contractually the borrower bears all the housing price risk since no ex ante provision exists for modifying the mortgage if housing prices fall. Recognizing that house price risk exists, lenders may require that borrowers purchase mortgage insurance, but this

typically only provides partial loss coverage in the event the borrower defaults.

Following the seminal works of Black and Scholes (1973) and Merton (1973), economist applied option pricing theory to mortgages.[3] Using option pricing models, researchers demonstrated that the mortgagor's right to prepay is the equivalent of a call option on the mortgage, and that the borrower's implicit ability to default on the mortgage is equivalent to a put option on the house. The mortgage pricing literature clearly shows that these options are valuable, that they interact with each other, and that they can be explicitly priced. Furthermore, theoretical papers such as Kau, et al, (1992) demonstrate that much of the observed default and prepayment behavior can be explained through models that assume that borrowers exercise their put and call options only when it is in their best interests.

One of the principal insights gained from the application of option pricing theory to mortgage contracts is that the traditional mortgage contract provides the borrower with the ability to hedge exposure to house price risk through default: if prices fall enough, the borrower can simply default and "sell" the house to the bank in exchange for release from the future promised mortgage payments.[4] Thus, under the current system lenders take on explicitly the risk of borrower default, and hence indirectly bear the risk that house prices will decline.[5,6]

The recent housing bubble experienced in the United States further confirms the critical option pricing theory insight regarding the importance of negative equity in motivating borrower default. For example, between 2000 and 2005, average house prices at the national level increased more than 5% per year and some local markets saw price increases of more than 20% per year followed by substantial price declines starting in 2006.[7] As a consequence, the number of mortgage holders with

---

[3]Early research applying option pricing techniques to mortgages include Buser and Hendershott (1984), Epperson et al. (1985), Foster and Van Order (1984), Kau et al. (1992, 1995), Schwartz and Torous (1992), and Titman and Torous (1989) among others. See Kau and Keenan (1995) for a survey of the extant literature.

[4]While it is true that in some States lenders could sue the borrower for a deficiency judgement, in many States lenders are explicitly prevented from pursuing deficiency judgments. Further, if a borrower has few other assets then even in those States where lender can pursue deficiency judgments, they will rarely bother to do so.

[5]Schwartz and Torous (1992) expand these conditions to note that default is exercised under a joint condition that the value of the house (1) is less than the present value of the mortgage payments and (2) is less than the outstanding principal amount. In addition, Kau and Kim (1994) explain why borrowers do not default exactly when the house value falls below the mortgage value by noting that the value of future default can be significant and can lead to decisions not to default in the present.

[6]We note that the present value of the mortgage includes the present value of defaulting or prepaying in the future - options given up if one defaults or prepays today. Further, the present value of default must also include any benefits that accrue from default above the simple cessation of payments. For example, these benefits include living in the house "rent free" during the foreclosure period.

[7]According to the August 2008 *RPX Monthly Housing Market Report*, 21 of the 25 MSAs followed by RPX lost at least 50% of the price increase they experienced from January 2004 to their respective price peaks in 2005 to 2007.

negative equity in 2008 hit historic levels. For example, Stempel (2008) reports that 7.63 million homeowners had negative equity in September 2008, or approximately 18% of all homeowners.[8] As a result of the rising number homeowners with mortgages that are "underwater", mortgage default and foreclosure rates have hit historic proportions, confirming the options-model notion that default is a function of downward price changes in housing.[9,10]

Currently, it is very difficult for lenders (or borrowers) to hedge against default risk. Although the relatively new futures market for house prices offers a limited ability to hedge housing markets in certain cities, the S&P Case-Shiller index-based derivatives are very costly to use as hedging instruments because of basis risk.[11] This basis risk stems from the effect of the interaction of house prices and interest rates on the options to default and prepay. In fact, the low trading volume associated with the S&P Case-Shiller indexes suggests that market participants do not find these contracts sufficient to hedge default risk on the traditional mortgage contracts.[12]

As a result of the difficulty in hedging default risk, mortgage contracts and the legal tradition surrounding them were designed to minimize the potential for borrower default.[13] The legal system imposes deadweight costs on both the borrower and the lender in order to provide incentives for both parties to avoid default. Indeed, if one assumes that mortgage contracts were designed to deter a moral problem, then it would make sense to make default as costly as possible.[14] In fact, the legal foreclosure process does exactly this. For example, Hatcher (2006) reports that a leading mortgage lender estimates that foreclosures cost over $50,000 per incident. Earlier analysis by

Furthermore, 8 MSAs lost all of the price appreciation experienced between January 2004 and their peak.

[8] Furthermore, the States with exceptionally large price declines (Arizona, California, Florida, Georgia, Michigan, Nevada, and Ohio), accounted for 64% of all mortgages with negative equity but only 41% of all mortgages.

[9] See Quercia and Stegman (1992), Vandell (1993), Kau and Keenan (1995), and LaCour-Little (2008) for surveys of the empirical default literature.

[10] For example, as of December 2008, statistics collected by the Mortgage Bankers Association reveal that 9.96% of all mortgages outstanding were either in delinquency (30 or more days late) or foreclosure (see "Delinquencies Increase, Foreclosure Starts Flat in Latest MBA National Delinquency Survey", MBA Press Release December 5, 2008, http://www.mbaa.org/NewsandMedia/PressCenter/66626.htm.) Another report indicated that in July 2008, the 60+ day delinquency rate on 2006 vintage prime fixed-rate mortgages was over 1.5% whereas the 60+ day delinquency rate on 2003 and 2004 vintage prime fixed-rate mortgages was less than 0.5% (Deutsche Bank, *RMBS Observer*, (July 21, 2008) page 18). Furthermore, approximately 25% of subprime mortgages were in default as of May 2008 (Bernanke (2008)).

[11] The Chicago Mercantile Exchange (CME) began trading the S&P Case-Shiller Metro Area Home Price Indices in 2006 for 10 metro areas.

[12] See Shiller (2008) for a discussion of various house price risk insurance schemes involving the use of CME contract.

[13] Jaffe and Sharpe (1996) provide an extensive discussion of the economic and legal theories regarding mortgage contracts.

[14] Jaffe and Sharpe (1996) explicitly note that certain legal scholars view mortgage contracts as moral promises and should be enforceable as such.

Ambrose and Capone (1996) using data from the early 1990s noted that typical direct costs to the lender associated with foreclosure (representing legal and sales expenses) were approximately 11 percent of the initial mortgage loan amount, in addition to the unrecovered loss on the loan balance. Finally, borrowers who default face higher credit costs in the future while lenders face uncertain costs over holding and selling the collateral property.

Instead of focusing on the "moral" aspect of the borrower's ability to pay, we propose a new mortgage contract that effectively minimizes the borrower's incentive resulting from declining house prices to exercise the embedded put option. Our new mortgage automatically resets the principle balance at various dates to the minimum of the originally scheduled balance or the value of the house, reducing the borrower's incentive to default if the house value declines.[15] We refer to this contract as the Adjustable Balance Mortgage (ABM).[16]

At origination, the ABM is like a fixed-rate mortgage in that it has a fixed contract rate, maturity term, and is fully amortizing. At fixed, pre-set intervals, the lender and the borrower determine the value of the house.[17] If the house value is lower than the then originally scheduled balance for that date, the loan balance is set equal to the house value, and the monthly payment is re-calculated based on this new value. If the house retains its initial value or increases in value, then the loan balance and payments remain unchanged just as in a standard fixed rate mortgage.

To illustrate the adjustment process, assume that the house price declined from origination to the first adjustment date and consider the following three scenarios at the second adjustment date: an upward movement in house prices, no change in house prices, or a further decline in house prices. In the first case, the house value increases between the first and the second adjustment dates. Thus, the balance is increased to the minimum of the new house value or the originally scheduled balance at the second adjustment date. As a result, the monthly payment goes up, but it does not exceed the mortgage payment specified at origination.

In the second case, the house value is exactly the same as at the first adjustment date. The balance is set to the minimum of the scheduled balance for that month or the house value. If

---

[15]The house value would be determined either through a Case-Shiller-Weiss (CSW) type index or other automated valuation system; individual appraisals at each adjustment date would not be feasible do to transactions costs.

[16]One could consider alternative adjustment schemes, such as resetting to maintain a percent-of-house-value ratio on specific dates, however, our modeling of this contract demonstrates that type of process is too costly to implement.

[17]We envision that the house value will be determined by applying the appreciation or depreciation from contract origination as revealed through a repeat-sales index.

the original amortization has reached the point where the balance would be less than the house value, then, of course, the payment is just the original payment. If we have not reached that point, however, then the balance is set to the house value and the payment recalculated - this means that from the first reset to the second reset date the balance would remain constant (actually, it would amortize each month, but then reset to the old value because the house valued remained constant.)

Finally, in case three, the house value declines between the first and second adjustment dates. Thus, the new balance is the new house price and the monthly payment is adjusted lower accordingly.

The ABM results in an explicit risk-sharing between the borrower and lender with respect to house prices. Before the loan balance is reset, the borrower will have lost whatever initial equity they had in the property, plus any equity that they would have built-up through the amortization process. Should the house price fall below the balance triggering a reset, and the house value then subsequently rises, the lender recovers their lost value first. In addition, if the house value rises above the originally scheduled balance on a reset date, then the owner begins to recover their equity as well. Note that the provision for the borrower to recover equity provides an economic incentive for the borrower to maintain the property even in the face of substantial price declines. Standard mortgages lack this economic incentive and evidence clearly shows that borrowers fail to maintain their properties when anticipating future default.

Our adjustable balance mortgage is similar in spirit to the "buy your own mortgage" (BYOM) innovation proposed by Hancock and Passmore (2008) and the "continuous workout mortgage" proposed by Shiller (2008). The BYOM is based on the Danish "mark-to-market" mortgage concept. As Svenstrup and Willeman (2006) point out, the Danish mortgage gives the borrower the option to prepay at the mortgage market value rather than at par (as in the U.S.). Thus, the Danish mortgage may reduce the incentive to default if the house values is less than the mortgage par value, as the borrower has the option to prepay at the lower market value. Borrowing from the Danish concept, Hancock and Passmore (2008) propose a mortgage that gives the borrower the right to pay the lender the proceeds from the house sale rather than the par value, in essence giving the borrower insurance against declines in the house value. However, the BYOM exposes the lender to significant moral hazard as the borrower has control over the underlying asset price. Although similar to the BYOM, the adjustable balance mortgage avoids this moral hazard by providing the

borrower with an automatic balance reset based on changes in an area wide movement in asset prices. The continuous workout mortgage advocated by Shiller (2008) is similar to the adjustable balance mortgage in that the mortgage balance is automatically adjusted to changes in a neighborhood (or area) house price index. However, unlike the adjustable balance mortgage, the continuous workout mortgage is designed to keep the borrower's equity constant. Thus, the mortgage balance is increased if home prices rise, limiting the capital accumulation feature of home ownership that is often advertised as a major benefit to home ownership.

3   A Formal Contingent Claims Model of the ABM

As is customary in the mortgage pricing literature, we develop a model of the ABM by utilizing the insights from Black and Scholes (1973) to note that in a perfect capital market the present value of a contingent claim (such as a mortgage contract)is:

$$V(r, H, T) = E\left[e^{-\int_0^T r(t)dt}\overline{V}(r, H, T)\right] \tag{1}$$

where $\overline{V}(r, H, T)$ is the terminal value of the mortgage contract expiring at T. The model contains two sources of uncertainty, interest rates (r) and house prices (H). We assume interest rates follow the Cox, Ingersoll, and Ross (1985) process:

$$d(r) = \gamma(\theta - r)dt + \sigma_r\sqrt{r}dz_r \tag{2}$$

where θ is the steady state mean rate, γ is the speed of adjustment factor, σ_r is the volatility of interest rates, dz_r is a standard Wiener process, and the local expectations hypothesis holds.

For the second risk factor, we assume the house price follows the standard stochastic process,

$$\frac{dH}{H} = (\alpha - s)\,dt + \sigma_H dz_H \tag{3}$$

where α is the total return to housing, s is the service flow, σ_H is the volatility of housing returns, and dz_H is a Wiener process. Assuming the correlation coefficient between dz_H and dz_r is ρ, then

equation (1) is the solution to the following partial differential equation (PDE)

$$\frac{1}{2}H^2\sigma_H^2\frac{\partial^2 X}{\partial H^2} + \rho H\sqrt{r}\sigma_H\sigma_r\frac{\partial^2 X}{\partial H\partial r} + \frac{1}{2}r\sigma_r^2\frac{\partial^2 X}{\partial r^2} + \gamma\left(\theta - r\right)\frac{\partial X}{\partial r} + \left(r - s\right)H\frac{\partial X}{\partial H} + \frac{\partial X}{\partial t} - rX = 0 \quad (4)$$

solved backwards through time.

Clearly the ABM is a highly-path dependent mortgage, and as such does not have a closed-form solution. As a result, we use a numerical method based on Nelson and Ramaswamy (1990) to value the mortgage and to solve for equilibrium contract rates. This method doubly transforms the two state variables, r and H, to allow for a simpler two-dimensional binomial-model.[18] We model the ABM and a standard fixed-rate mortgage (FRM) within this bivariate-binomial lattice.

3.1 Boundary Conditions at Maturity (T)

Given that we can model the evolution of r and H, we must define the appropriate boundary conditions for the FRM and the ABM. Since the model uses backwards-induction, we begin with the boundary conditions at the terminal time. Consider first the terminal boundary conditions for the FRM with an annual contract rate r_c and where the monthly mortgage payment (PMT_t) is given by:

$$PMT_t = Balance_0 * \left[\frac{r_c/12}{1 - \frac{1}{(1+r_c/12)^t}}\right], \quad (5)$$

for $t = 1$ to T. At termination, the present value of the promised payments, A_T, is given as:

$$A_T = PMT_T. \quad (6)$$

Furthermore, the values of default, prepayment, and the mortgage are given as:

$$D_T = max[0, PMT_T - H_T] \quad (7)$$

$$C_T = 0 \quad (8)$$

$$V_T = A_T - C_T - D_T. \quad (9)$$

---

[18]This numerical technique is well established in the mortgage literature and is fully described in several papers including Ambrose, Buttimer, and Capone (1997), and Hilliard, Kau, and Slawson (1998).

The terminal boundary conditions for the ABM are similar since the borrower only faces one decision at the terminal time step, make the final payment or default. However, the ABM is operationally much more complex because the payment amount is not fixed, but rather is conditional upon the balance remaining after the last reset date, denoted as $T - T_R$, where T_R is the number of months following the last reset date.

At time T, we do not know with certainty the balance at the last reset date because we do not know the path of house values through time. We do, however, know the upper bound, which is the originally scheduled balance for that reset date. Because of the structure of the bivariate binomial lattice, we also know that only a finite number of potential mortgage balances exist as of that reset date.[19] From the potential balances at $T - T_R$, we then determine the finite number of potential balances that exist at termination T.

Thus at time T, we solve for every possible mortgage payment value and then solve for the terminal values for default, prepayment, the promised mortgage payment, and the value of the mortgage conditional upon the payment value. Formally, we make the ABM payment conditional upon the balance at the previous resetting date, $Balance_{last}$.[20] This is given by:

$$PMT_{T|Balance_{last}} = Balance_{last} * \left[\frac{r_c^*/12}{1 - \frac{1}{(1+r_c^*/12)^{T_R}}} \right], \tag{10}$$

where r_c^* represents the ABM contract interest rate set at origination. The boundary conditions for the ABM are:

$$A_{T|Balance_{last}} = PMT_{T|Balance_{last}} \tag{11}$$

$$D_{T|Balance_{last}} = max[0, PMT_{T|Balance_{last}} - H_T] \tag{12}$$

$$C_{T|Balance_{last}} = 0 \tag{13}$$

$$V_{T|Balance_{last}} = A_{T|Balance_{last}} - C_{T|Balance_{last}} - D_{T|Balance_{last}}. \tag{14}$$

As we work backwards in time we eventually get to time $T - T_R$ at which point we discard the

---

[19]As only a finite number of nodes exist in the lattice, only a finite number of paths and hence a finite number of potential balances could have existed at time $T - T_R$.

[20]To be completely accurate, the value is conditional upon the balance at $T - T_R$, which is itself conditional upon the path that has been taken to get to time $T - T_R$. For expositional clarity, we suppress the notation for this second level of conditionality.

extraneous values. Of course, the actual balance at time $T - T_R$ is itself a function of the balance at time $T - 2 * T_R$. However, the uncertainty does partially resolve at each reset date, providing sufficient resolution of the uncertainty to allow us to work our way back to time 0, at which point all uncertainty resolves. The uncertainty applies to all calculations that are dependent upon the balance of the loan, including payments, the default option, and the prepayment option.

3.2 Boundary Conditions Prior to Maturity $(t < T)$

For both the FRM and the ABM at payment dates other than the terminal date, the borrower chooses between one of three actions. First, the borrower can make the schedule payment, thus continuing the mortgage. Second, the borrower can terminate the mortgage via prepayment. Third, the borrower can terminate the mortgage through default. For the FRM, the present value of the future promised payments is:

$$A_t = PMT_t + \frac{E[A_{t+1}]}{(1 + r_t)}, \tag{15}$$

where $E[A_{t+1}]$ is the expected value of A at the next time step.[21] The payoff to continuation is:

$$V_t = A_t - C_t - D_t, \tag{16}$$

where C_t is equal to the expected present value of the future prepayment option, and D_t is equal to the expected present value future default.

The payoff to immediate prepayment is:

$$C_t = A_t - Balance_t. \tag{17}$$

Of course, if the borrower prepays at t, the borrower cannot default in the future, and so $D_t = 0$. The payoff to immediate default is:

$$D_t = A_t - H_t. \tag{18}$$

---

[21]Note that even with the FRM, we have to consider the expected value of A_{t+1} because of the stochastic nature of r_{t+1}.

Obviously, if the borrower defaults at t, then they cannot prepay in the future, so $C_t = 0$.

The lender receives V_t if the borrower makes the scheduled payment, the outstanding balance ($Balance_t$) if the borrower prepays, or the house value (H_t) if the borrower defaults. The borrower will select the option that maximizes their wealth, or that equivalently minimizes the wealth of the lender. Thus, the value of loan to the lender is given as:

$$V_t = min(A_t - C_t - D_t, Balance_t, H_t). \tag{19}$$

For the ABM, the borrower applies the same logic. That is, the borrower minimizes the wealth of the lender in order to maximize their wealth. The difference is that all of the equations that are dependent upon the balance of the loan at the last reset date must be noted as being conditional values. Thus,

$$A_{t|Balance_{last}} = PMT_{t|Balance_{last}} + \frac{E[A_{t+1|Balance_{last}}]}{(1 + r_r)} \tag{20}$$

is the present value of the promised stream of payments, conditional on the balance at the last resetting date ($Balance_{last}$). The value of continuation therefore is:

$$V_{t|Balance_{last}} = A_{t|Balance_{last}} - C_{t|Balance_{last}} - D_{t|Balance_{last}}, \tag{21}$$

where the conditional values C and D are still the expected present value of the prepayment and default options in the future, conditional upon the balance at the last reset date. Similarly, the payoff to the immediate default and prepayment options must be expressed as being conditional upon the balance at the last reset date:

$$C_{t|Balance_{last}} = A_{t|Balance_{last}} - Balance_{t|Balance_{last}}, \tag{22}$$

and

$$D_{t|Balance_{last}} = A_{t|Balance_{last}} - H_{t|Balance_{last}}. \tag{23}$$

Once again, if the borrower defaults at t, then they cannot default in the future so $C_{t|Balance_{last}}$ is set to zero, and if the borrower prepays today then the value of default $D_{t|Balance_{last}}$ is set to zero. Finally, from the lender's perspective, the value of the loan to the lender is:

$$V_{t|Balance_{last}} = min(A_{t|Balance_{last}} - C_{t|Balance_{last}} - D_{t|Balance_{last}}, Balance_{t|Balance_{last}}, H_t). \quad (24)$$

We also note that at any interior point other than a payment due-date, the borrower does not have to take any action should they elect to default. Their default decision will only become apparent at the next payment due-date. As a result, at the non-payment due dates, the boundary conditions do not consider the case of immediate default.

4 Numerical Analysis

As mentioned above, the path dependency inherent in the ABM precludes a closed form solution and thus we use numerical analysis to solve the equilibrium contract rates and option values. Table 1 summarizes the input parameters used in the numerical analysis. These parameter values are generally consistent with the input parameters used in the literature. We begin the analysis by presenting in Table 2 results showing the equilibrium contract for each mortgage type (FRM and multiple reset-option ABMs) under three common loan-to-value ratio assumptions (80%, 90%, and 95%.) In order to highlight the costs associated with the ABM, we present four alternative versions. First, we present an annual adjusting contract (the balance is adjusted annually at the mortgage origination anniversary). Second, we demonstrate the impact of more frequent adjustments by pricing a quarterly adjusting contract. Third, we show the pricing for two contracts that reset only once during the mortgage life (at month 36 and month 60). Obviously, many other adjustment date variations are possible and the alternatives shown in Table 2 are representative.[22]

The results in each Panel in Table 2 are in equilibrium. That is, we solved for the contract rate for each mortgage which equates the value of the mortgage (V) with the cash received at time origination. We assume that the borrower pays 1.5 points at origination and thus the equilibrium "par" value of the mortgage is $98.50. We also present the equilibrium origination values for the

---

[22]For example, one could consider a mortgage that gives the borrower the option to demand a reset at any time prior to maturity.

present value of the payments (A), the value of the borrower's prepayment option (C), and the value of the default option (D).

We note that two major results or implications come from Table 2. First, the equilibrium contract rates for the ABM variants are typically 25 to 40 basis points higher than for the standard FRM. For example, in Panel C (the 95% LTV assumption), we note that the equilibrium FRM contract rate is 6.22% while the annual ABM and quarterly ABM have equilibrium contract rates of 6.46 percent and 6.64%, respectively. Thus, the quarterly ABM has an equilibrium contract rate that is 42 basis points greater than the standard FRM contract. In contrast, the premium required for the quarterly ABM under the 80% LTV assumption (Panel A) is only 13 basis points and the premium under the 90% LTV case (Panel B) is 23 basis points. Intuitively, as the LTV increases, the borrower's initial equity position declines and the probability of the collateral value falling below the mortgage balance increases. As a result, the ABM is more valuable to the borrower for higher LTV contracts and hence the equilibrium contract rate premium increases to compensate the lender for bearing the additional house price risk.

Since the ABM is designed to shift house price risk from the borrower to the lender, Table 2 shows that the ABM dramatically reduces the financial value of the default option, as expected. For example, the annual ABM reduces the value of default by approximately 36% (from 4.54 to 2.9 in the 95% LTV case) while the examples with one-time adjustments at months 36 and 60 only reduce the default value by about 19% and 25%, respectively. However, moving from the annual adjustment to a quarterly adjustment completely eliminates the financial (or rational) incentive for default. Thus, the quarterly ABM is more effective at reducing the value of (i.e. future economic incentive for) default than any of the others.

The second major result evident in Table 2 is that the value of the prepayment option goes up substantially for the ABM contracts. However, the prepayment option value increase is not due to the increase in the equilibrium contract rate, but rather because of the fact that default is no longer a competing source of termination. Although the equilibrium contract rate is higher for the ABM mortgages than for the FRM mortgage, the difference across contract rates is not large. Thus, the factor driving the increase in prepayment is the fact that under the ABM, the borrower does not have the competing risk of default. As a result, borrowers will prepay more rapidly. Furthermore, faster expected prepayment under the ABM is good news from a lender's hedging standpoint since

prepayments will be more tightly tied to interest rate changes.

To see the effect on prepayment, consider Figures 1 and 2 that show the borrower's default, prepayment, and continue regions in state space (house price and interest rate) 36 months after origination for the standard FRM and quarterly ABM, respectively.[23] In essence, the state space figures show the interaction of house prices and interest rates on the borrower's decision to continue or terminate the mortgage. For example, the green region in Figure 1 shows the default region for a borrower in a standard FRM at month 36. As expected, default is optimal when house prices have declined (less than 1.0) and spot interest rates are relatively low (less than 15 percent). The red region reveals the combination of house prices and interest rates where the borrower would optimally prepay the mortgage at month 36 and the blue region represents the combinations of house prices and interest rates where the borrower would find it optimal to make the next payment.[24] In comparison, Figure 2 shows the borrower's termination regions at month 36 for the quarterly ABM. Consistent with the results presented in Table 2, the green default region is not present in Figure 2 indicating that default is not an optimal decision for the borrower. However, Figure 2 does show the increase in the value of prepayment as certain regions of the state space where house prices and interest rates are low are dominated by prepayment.

As noted above, the equilibrium contract rates for the various ABM contracts are approximately 25 to 40 basis points from the FRM base case. Thus, we consider whether the increase in the equilibrium contract rate is low enough to induce borrowers to prefer the ABM contract over the standard FRM contract. In other words, how much would a borrower be willing to pay, in the form of a higher contract interest rate, to eliminate the potential for future negative equity?[25] In order to answer this question, we recognize that, under the ABM, mortgage insurance would be either unneeded or priced at such a low rate (to cover frictional, non-financial defaults) that the net cost

---

[23]Recall that we use a Nelson-Ramaswamy double transformation that involves both a log-transformation and then a combination of the log-transformed variables. The doubly-transformed state space is, as one would expect, rectangular in shape. To present the information in a meaningful manner, however, we undid these transformations in the figures, resulting in the triangular shape. However, the graph is not probability weighted, that is it shows the borrower's decision at every node in the state space regardless of the probability of reaching that node. The nodes where $r > 10\%$ or where $H > 1.5$ have very low probability of being reached. The vast majority of the probability mass is bounded by $.01 < r < .10$ and $.5 < H < 1.5$.

[24]State space graphs can be produced for any point over the life of the mortgage. Obviously, the default, prepay, and continue regions will shift through time. We selected month 36 in Figures 1 and 2 to illustrate the interactions of the borrower options to terminate the mortgage.

[25]A similar question centers around the borrower's choice between adjustable-rate (ARM) contracts and fixed-rate (FRM) contracts.

to the borrower might actually be less than under an FRM. Another way of thinking about the issue of contract desirability is to ask, if the lender kept the contract rate fixed at the current FRM rate, how many points would the borrower have to pay for this loan? In Table 3 we answer this question by holding the interest rate constant at the standard FRM contract rate that balances the contract (assuming the borrowers pays 1.5 points) and varying the total points paid at origination. For example, in Table 3 Panel C (the 95% LTV mortgage), we assume the mortgage interest rate is 6.2170% for all contracts. We see that for the annual adjusting ABM, the borrower would have to pay 42 basis points more in up-front interest (1.93 points versus 1.5 points under the FRM). For the quarterly adjusting contract that eliminates financially induced default, the borrower would have to pay 71 basis points more than under the standard FRM contract. In contrast, typical private mortgage insurance (PMI) contracts require the payment of 0.5% (50 basis points) of the loan balance each year while the Federal Housing Administration (FHA) charges an up-front fee of 1.5% (150 basis points) of the loan amount plus a monthly fee. Clearly the additional costs associated with the quarterly adjusting ABM are below the standard costs associated with PMI and substantially lower than the costs associated with FHA insurance.

Our model also allows us to consider the impact of uncertainty, as reflected in the volatility assumptions underlying the stochastic state processes, on the default values and equilibrium interest rates. Thus, in table 4 we report the equilibrium contract values corresponding to the standard mortgage, the annual adjusting mortgage, and the quarterly adjusting mortgage for combinations of house price volatility (σ_H) and interest rate volatility (σ_r). Table 4 reports the results assuming loan-to-values ratios at origination of 80%, 90%, and 95% for the following volatility combinations: [$\sigma_H = \sigma_r = 5\%$], [$\sigma_H = 5\%, \sigma_r = 15\%$], [$\sigma_H = 15\%, \sigma_r = 5\%$], and [$\sigma_H = \sigma_r = 15\%$]. As before, we price the mortgages in equilibrium with a par value of \$98.50. Turning to Panel C (LTV=95%), we find several interesting features. Consistent with option pricing theory, increasing the underlying volatility of the state processes increases the value of the embedded options to default and prepay. For example, looking at the annual adjusting ABM, we see that increasing σ_r by a factor of three from 5% to 15% (holding σ_H constant at 5%) increases the value of prepayment from 3.15 to 19.75 and the value of default from 0.14 to 0.57, respectively. Interestingly, increasing the house price volatility by a factor of three has an even greater effect on prepayment than the corresponding increase in interest rate volatility. For example, increasing σ_H from 5% to 15% (holding σ_r constant

15

at 5%) increases the value of prepayment from 3.15 to 22.11 and the value of default from 0.14 to 6.27, respectively.

Table 4 also highlights the highly complex interaction between prepayment and default depending upon the assumption regarding the state process volatility. For example, consider the results for the standard mortgage contract. Across all LTV assumptions, we see that increasing the house price volatility (σ_H) increases the value of the default option. For example, looking at the 95% LTV case, we see that increasing σ_H from 5% to 15% (holding σ_r constant at 5%) increases the default value from 0.19 to 10.03. However, for the quarterly ABM, the same increase in σ_H actually causes the value of default to decrease from 0.06 to 0.00. In other words, default become less likely under the quarterly ABM when house price volatility is high.

5 Default Transaction Costs

Although we find the difference in equilibrium contact rates to be small, that is a matter of subjective opinion. Thus, we now focus on the question of whether reasonable circumstances exist where the ABM contract would have a lower contract rate than a standard FRM? If we assume that the lender faces unrecoverable transactions costs associated with default, then the answer is unequivocally affirmative. Lenders do clearly face such costs. For example, consider that lenders normally hire a real estate agent to sell the foreclosed property, at a typical cost of 6% in most parts of the US. In addition, lenders face other costs including insurance, maintenance, property taxes, and/or utilities while the house is being marketed. Based on these costs, some have estimated that transactions cost may exceed 30% of the outstanding loan balance.[26] Note that in our context, transaction costs include only the costs associated with the delinquency and foreclosure process, and not the loss in value from the house itself.

We modify our model to incorporate lender transaction costs.[27] As our goal is to fully reflect the worst case for the lender when calculating the equilibrium contract rates, we let the lender face default transaction costs but not the borrower. As a result, this creates a wedge in the mortgage value between the borrower and lender at origination. In order to compensate the lender for the

---

[26]See Pence (2003), Capone (1996), and Clauretie and Herzog (1990) for estimates of default and foreclosure costs.

[27]Other research has focused on borrower transaction costs and have assumed no cost for the lender (e.g. Kau and Slawson 1998, and Kau, Keenan, and Kim 1993).

anticipated costs if default occurs, all contract rates are higher. As a result, prepayment is higher in general. In order to demonstrate the effect of various levels of transaction costs, we calculated the equilibrium contract rate at every level of lender transaction cost from 0 to 10% (of original house value.) Figure 5 plots the equilibrium contract rates for lender default transaction cost levels for each of the mortgage contracts assuming a 95% LTV. Figure 5 clearly shows that if lender default transaction costs are very low (less than 3% of the original house value), then the standard FRM contract has the lowest contract interest rate. However, at higher levels of lender default costs (greater than 4%), the semiannual and quarterly adjusting ABM contracts have equilibrium contract rates less than the standard fixed-rate mortgage. In other words, under any reasonable assumption of non-recoverable lender default losses, the quarterly adjusting ABM contract would have a lower interest rate to the borrower than the current standard FRM for high LTV mortgages.

The results are less intriguing at an 80% LTV, where our analysis indicates that the FRM always has a lower equilibrium contract rate than the ABM variations. However, this result is largely determined by the fact that default is relatively rare with an 80% LTV mortgage. For example, in Table 2 we saw that the value of default was approximately 28 basis points for the FRM option and 13 basis points for the annual ABM contract. In addition, the defaults, when they do happen, tend to occur later in the life of the mortgage since it takes time for the house value to fall below the mortgage balance in order to trigger an optimal default. As a result, present value discounting effects also come into play. Nevertheless, it is the case that at 80% LTV, the FRM always has a lower equilibrium contract rate than the ABM suggesting that the ABM contract would be preferred by borrowers seeking higher LTV contracts.

6 Policy Implications: One-time Mortgage Modification Programs

As indicated in Section 2, individuals who purchased a house or refinanced a mortgage in 2004 and 2005 at the peak of the recent housing market bubble encountered significant price declines through 2008 with further price declines expected through at least 2010. The embedded put option in these mortgages is clearly "in-the-money" for many of these borrowers (that is, the underlying asset value is below the present value of the debt contract – a condition popularly referred to as "negative equity".) Thus, it is not surprising that we now face significant mortgage defaults.

In response to this foreclosure crisis, a number of institutions have developed programs or policies that attempt to mitigate the effects of declining house values. For example, the FDIC Loan Modification Program, first implemented at IndyMac Federal Bank in August 2008, is an attempt to reduce foreclosures and bring stability to the housing and mortgage markets by re-writing the terms on troubled mortgages. Rather than exclusively altering the loan amount, the program focuses on monthly payments by changing the mortgage interest rate such that the borrower receives an immediate payment relief of 10% with the mortgage principal, interest, taxes, and insurance (PITI) payment not to exceed 38%. The program provides for housing-to-income targets as low as 31% in order to meet the target 10% payment reduction. In addition, the FDIC program allows for partial principal reductions through forbearance. However, all principal reductions are due in the future after the remaining principal is amortized. Finally, the modified loans are analyzed to determine that the costs associated with any modifications are less than the estimated costs of foreclosure. However, consistent with the predictions from mortgage options models, the FDIC modification plan has achieved limited success as it fails to address the house price risk factor driving mortgage defaults.

The Office of the Comptroller of the Currency (OCC) reports that as of the third quarter of 2008, 133,000 mortgage had been modified.[28] Unfortunately, the success rate for these modifications is low as the OCC reports that up to 37% of these loans were 60 days into a second default within 6 months of modification. Several potential reasons exist for the failure of these modifications, including: moral hazard, lowered effective cost of default, and mis-estimated house value. We argue that the real problem is that the new LTVs are essentially 100%, creating a high probability of the loan default option being "in-the-money" within a short time after modification.

To see the implications of modifying loans to 100% LTV, consider the borrower's optimal actions at month 36 and 42 under the annual ABM model as reflected in Figures 3 and 4. The figures reveal the optimal borrower action (default, prepay, continue the mortgage) in the state space (combinations of house price and market interest rate) conditional on the annual adjustable balance mortgage surviving to months 36 and 42, respectively. Since the mortgage considered adjusts annually on the anniversary date of origination, month 36 coincides with a possible balance reset at

---

[28]Many of these modifications (but not all) were ARMs converted to FRMs with the balance reset to the house value.

month 36, while month 42 is 6 months past the last reset and 6 months away from the next reset. Thus, at month 36 (Figure 3), we see that the annual adjusting ABM has removed default from the possible actions considered by the borrower. Figure 3 shows that under all possible combinations of house values and interest rates at month 36, the borrower either prepays or continues the mortgage as the borrower has no financial incentive to default since the mortgage balance is reset to the minimum of the house value or the scheduled amortized loan balance. However, 6 months later, Figure 4 reveals that borrower default is now a distinct possibility. The green shaded region in Figure 4 corresponds to the combination of house values and interest rates that would lead the borrower to optimally default. Thus, even with an annual balance adjustment, we see that borrower default remains a possibility.

Clearly then, mortgage modification proposals that call for a one-time principal adjustment do not remove the potential for future financially motivated default. For a true one-shot mortgage, like the loan modification plan being proposed by the Treasury, the Federal Reserve, and FDIC, the borrower incentives to default are even higher as these plans do not require principal reductions but focus on borrower payment-to-income. The only way to truly reduce the default probability is either: reset the mortgage balance to a LTV that is lower than 100%, probably around 80%, or have frequent, predictable balance resets (as demonstrated in the quarterly ABM). The balance resets must be sufficiently frequent to induce the borrower to not default even if the house price dips. The key implication is that the programs being contemplated by regulatory authorities (notably the Federal Reserve and the FDIC) will not significantly reduce defaults unless house prices rapidly stabilize or go up, independent of issues such as moral hazard.

7   Conclusion

Following the tradition that economic crises often spur financial innovations, we propose a new fixed-rate mortgage contract that mitigates the risks associated with downward movements in house prices. Similar to the adjustable-rate mortgage that shifts interest rate risk from the lender to the borrower, our new adjustable balance mortgage (ABM) shifts house price risk from the lender to the borrower. The ABM automatically resets the principle balance at various dates to the minimum of the originally scheduled balance or the value of the house.

19

Using a standard bivariate-binomial lattice model, we demonstrate numerically that the value of the borrower's default option declines as the frequency of balance resets increases. Our analysis reveals that the borrower's default option is effectively eliminated under a quarterly balance adjusting mortgage. Since we solve the contract values in equilibrium, our analysis shows that for a 95% LTV mortgage, the cost to the borrower for the quarterly ABM, in the form of a higher contract interest rate, is 42 basis points relative to the standard fixed-rate contract. However, under any scenario assuming reasonable dead-weight losses to the lender from default, our analysis demonstrates that the ABM has lower contract rates than the standard fixed-rate mortgage when the loan-to-value ratio is above 80%.

As previously mentioned, the interaction of the default and prepayment options in a fixed rate mortgage make it very difficult to hedge default risk using any of the currently existing property derivatives. The same problem plagues prepayment hedging, although until recently default was so rare (and had so little value) that it did not preclude interest rate hedging. However, under the ABM, lenders now have an incentive to use a derivative contract, like the existing CME house price contracts and the option contracts, in particular, to hedge against the risk of the house value declining. For example, consider a lender (or investor) holding a portfolio of mortgages originated in Phoenix. Since the lender is exposed to the systematic risk of the overall Phoenix housing market (as reflected in the changes in the broad Phoenix housing index), a natural incentive now exists to use the CME Phoenix futures contract to hedge this risk.

Finally, our analysis reveals why current loan modification proposals that do not reduce principal balances have significant redefault rates. Using a one-time adjustment as an example, we show that the value (and hence probability) of default remain significant six months after the balance reset. As a result, unless housing markets stabilize or house prices return to bubble levels, the current mortgage modification plans will face significant re-defaults. Rather, using our model, lenders could effectively price asset value driven future default probabilities and refinance troubled borrowers into the ABM contract that eliminates the incentives for optimal or ruthless default.

8   References

Ambrose, Brent W., and Richard J. Buttimer, Jr., "Embedded Options in the Mortgage Contract," *The Journal of Real Estate Finance and Economics,* 21:2 (2000) 95-111.

Ambrose, Brent W., Richard J. Buttimer, Jr., and Charles A. Capone, Jr., "Pricing Mortgage Default and Foreclosure Delay," *Journal of Money, Credit, and Banking,* 29:3 (August 1997) 314-325.

Ambrose, Brent W. and Charles A. Capone, Jr., "Cost-Benefit Analysis of Single-Family Foreclosure Alternatives," *Journal of Real Estate Finance and Economics,* 13: (1996) 105-120.

Ambrose, Brent W. and Charles A. Capone, Jr., "The Hazard Rates of First and Second Default," *The Journal of Real Estate Finance and Economics,* 20:3 (2000) 275-293.

Ambrose, Brent W., Charles A. Capone, Jr., and Yongheng Deng, "Optimal Put Exercise: An Empirical Examination of Conditions for Mortgage Foreclosure," *Journal of Real Estate Finance and Economics,* 23:2 (2001) 213-234.

Bernanke, Ben S. "Mortgage Delinquencies and Foreclosures" Columbia Business School's 32nd Annual Dinner, New York, New York (2008-05-05). Retrieved on 2008-05-19.

Black, Fisher, and Myron Scholes. "The Pricing of Options and Corporate Liabilities." *Journal of Political Economy* 81 (1973), 637-54.

Buser, S.A. and P.H. Hendershott. "Pricing Default-Free Fixed Rate Mortgages," *Housing Finance Review* (1984) 3:4 405-429.

Capone, Jr., C.A. *Providing Alternatives to Mortgage Foreclosure: A Report to Congress.* Washington, D.C.: U.S. Department of Housing and Urban Development. (1996).

Capozza, Dennis R., Dick Kazarian, and Thomas A. Thomson. "The Conditional Probability of Mortgage Default," *Real Estate Economics,* 26:3 (1998), 359-390.

Clauretie, T.M. and T. Herzog. "The effect of state foreclosure laws on loan losses: Evidence from the mortgage insurance industry," *Journal of Risk and Insurance* 56:3 (1990), 221-233.

Cox, John C., Jonathan E. Ingersoll, Jr. and Stephen A. Ross. "An Intertemporal General Equilibrium Model Of Asset Prices," *Econometrica* 53:2 (1985), 363-384.

Epperson, J.F., J.B. Kau, D.C. Keenan, and W.J. Muller III. "Pricing Default Risk in Mortgages." *AREUEA Journal* (1985) 13:3 152-167.

Foster, C., and R. Van Order. "An Option Based Model of Mortgage Default." Housing Finance Review 3 (1984), 351-72.

Hancock, D. and w. Passmore, "Three Initiatives Enhancing the Mortgage Market and Promoting Financial Stability," *The B.E. Journal of Economic Analysis & Policy*: Vol. 9 : Iss. 3 (Symposium), (2009) Article 16.

Hatcher, D. "Foreclosure Alternatives: A Case for preserving Homeownership." *Profitwise News*

and Views (February 2006), 1-4

Hilliard, Jimmy, James B. Kau, and Carlos Slawson. "Valuing Prepayment and Default in a Fixed Rate Mortgage: A Bivariate Binomial Pricing Technique." *Real Estate Economics* 26:3 (1998), 431-468.

Hilliard, Jimmy E., Adam L. Schwartz, and Alan L. Tucker. "Bivariate Binomial Options Pricing With Generalized Interest Rate Processes." *Journal of Financial Research* 19:4 (1996), 585-602.

Herzog, J.P. and J.S. Early. "Home Mortgage Delinquency and Foreclosure." New York: National Bureau of Economic Research. (1970).

Jackson, J., and D. Kasserman. "Default Risk on Home Mortgage Loans: A Test of Competing Hypotheses." *Journal of Risk and Insurance* 3 (1980), 678-90.

Kau, James B. and Donald C. Keenan. "An Overview of the Option-Theoretic Pricing of Mortgages." *Journal of Housing Research* 6:2 (1995), 217-244.

Kau, James B., Donald C. Keenan, Taewon Kim. "Transaction Costs, Suboptimal Termination, and Default Probabilities." *Journal of the American Real Estate and Urban Economics Association* 21:3 (1993), 247-264.

Kau, James B., Donald C. Keenan, Taewon Kim. "Default Probabilities for Mortgages." *Journal of Urban Economics* 35:3 (1994), 278-296.

Kau, James B., and Taewon Kim. "Waiting to Default: The Value of Delay." *Journal of the American Real Estate and Urban Economics Association* 22:3 (1994), 539-551.

Kau, James B., Donald C. Keenan, Walter J. Muller, III, and James F. Epperson. "A Generalized Valuation Model for Fixed-Rate Residential Mortgages." *Journal of Money, Credit, and Banking* 24 (1992), 279-299.

Kau, James B., Donald C. Keenan, Walter J. Muller, III, and James F. Epperson. "Option Theory and Floating-Rate Securities with a Comparison of Adjustable- and Fixed-Rate Mortgages." *Journal of Business* 66 (1993), 595-618.

Kau, James B., Donald C. Keenan, Walter J. Muller, III, and James F. Epperson. "The Valuation at Origination of Fixed Rate Mortgage with Default and Prepayment." *Journal of Real Estate Finance and Economics* (1995) 11(1) 5-39.

LaCour-Little, M. "Default and Prepayment Risk in Residential Mortgage Loans: A Review and Synthesis." (2007) in S. Agarwal and B.W. Ambrose, editors. *Household Credit Usage: Personal Debt and Mortgages* Palgrave/MacMillan: New York, NY.

Merton, R. "Theory of Rational Option Pricing," *Bell Journal of Economics* (1973).

Pence, K.M. "Foreclosing on Opportunity: State Laws and Mortgage Credit," Board of Governors of the Federal Reserve System (May 13, 2003).

Quercia, R., and M.A. Stegman. "Residential Mortgage Default: A Review of the Literature." *Journal of Housing* 3 (1992), 341-380.

Quigley, John M., and Robert Van Order. "Explicit Tests of Contingent Claims Models of Mortgage Default," *Journal of Real Estate Finance and Economics* 11:2 (1995), 99-118.

RPX Monthly Housing Market Report, Radar Logic (October 31, 2008 www.radarlogic.com).

Schwartz, Edwardo S. and Walter N. Torous. "Prepayment, Default, and the Valuation of Mortgage Pass-through Securities." *Journal of Business* 65:2 (1992), 221-239.

Shiller, R.J. "Policies to Deal with the Implosion in the Mortgage Market," *The B.E. Journal of Economic Analysis & Policy*: Vol. 9 : Iss. 3 (Symposium), (2009) Article 4.

Shiller, R.J., *The Subprime Solution*, Princeton, NJ: Princeton Press. (2008).

Stempel, J., (2008), "One in five U.S. mortgage-holders underwater," *Reuters* (October 31, 2008)

Svenstrup, M. and S. Willeman. "Reforming Housing Finance: Perspectives from Denmark." *Journal of Real Estate Research* 28:2 (2006), 105-128.

Titman, S. and W.N. Torous. "Valuing Commercial Mortgages: An Empirical Investigation of the Contingent-Claims Approach to Pricing Risky Debt." *Journal of Finance* (1989) 44:2 345-373.

Vandell, K.D. "Handing Over the Keys: A Perspective on Mortgage Default Research." *Journal of the American Real Estate and Urban Economics Association* (1993) 21:3 211-46.

von Furstenberg, G. "Default Risk on FHA-Insured Home Mortgages as a Function of the Term of Financing: A Quantitative Analysis." *Journal of Finance* 24 (1969), 459-77.

von Furstenberg, G., and R.J. Green. "Home Mortgages Delinquency: A Cohort Analysis." *Journal of Finance* 29 (1974), 1545-48.

Williams, A.O., W. Bernake, and J. Kenkel. "Default Risk in Urban Mortgages: A Pittsburgh Prototype Analysis." *AREUEA Journal* 2 (1974), 101-12.

Table 1: **Base Case Parameters for Model**

Panel A: Model and General Economic Condition Parameters

Interest Rate Parameters:

Initial spot interest rate (r_0)	4%
Interest rate mean reversion adjustment factor (γ)	0.25
Steady state mean interest rate (θ)	6%
Interest rate volatility (σ_r)	10%

Housing Parameters:

Normalized house value at contract origination (H_0)	1.0
Housing service flow (S)	2%
House price volatility (σ_H)	10%

General Parameters:

Interest rate and housing price correlation (ρ)	0
Time steps per month used in numerical analysis	5

Panel B: Mortgage Specific Parameters

Loan-to-Value (LTV)	80%, 90%, 95%
Mortgage Term	360 months
Points Paid at Origination	1.5%
ABM Reset Frequency:	Once at month 60
	Once at month 36
	Annual
	Quarterly

Note: This table reports the input parameter values used in the numerical analysis.

Table 2: **Base Case Equilibrium Results (expressed as a percentage of the original loan balance)**

	Standard Mortgage	Annual Adjusting Mortgage	Quarterly Adjusting Mortgage	Reset Once at 36 Months	Reset Once at 60 Months
Panel A: 80% LTV					
Contract Rate	5.73%	5.86%	5.86%	5.85%	5.85%
Mortgage Value (V)	98.50	98.50	98.50	98.50	98.50
Present Value of payments (A)	107.87	108.77	108.85	108.76	108.62
Prepayment Option Value (C)	9.09	10.14	10.35	10.10	9.95
Default Option Value (D)	0.28	0.13	0.00	0.17	0.16
Panel B: 90% LTV					
Contract Rate	5.90%	6.08%	6.13%	6.05%	6.03%
Mortgage Value (V)	98.50	98.50	98.50	98.50	98.50
Present Value of payments (A)	109.96	110.95	111.48	110.34	110.14
Prepayment Option Value (C)	9.66	11.58	12.98	10.74	10.40
Default Option Value (D)	1.80	0.87	0.00	1.10	1.23
Panel C: 95% LTV					
Contract Rate	6.22%	6.46%	6.64%	6.39%	6.37%
Mortgage Value (V)	98.50	98.50	98.50	98.50	98.50
Present Value of payments (A)	113.72	115.03	117.13	113.60	113.55
Prepayment Option Value (C)	10.68	13.62	18.63	11.72	11.36
Default Option Value (D)	4.54	2.90	0.00	3.38	3.69

Note: This table shows the equilibrium contract values for the FRM and variations of the ABM under three common loan-to-value (LTV) ratio assumptions. The numerical analysis solves for the contract rate for each mortgage that equates the value of the mortgage (V) with the cash received at origination. We assume the borrower pays 1.5 points at origination resulting in an equilibrium "par" value of the mortgage of $98.50. The table also reports the equilibrium origination values for the present value of the mortgage payments (A), the value of the borrower's prepayment option (C), and the value of the default option (D).

Table 3: **Comparison of FRM and ABM Contracts Assuming a Constant Contract Rate**

	Standard Mortgage	Annual Adjusting Mortgage	Quarterly Adjusting Mortgage	Reset Once at 36 Months	Reset Once at 60 Months
Panel A: 80% LTV					
Points	1.50	1.75	1.77	1.75	1.73
Mortgage Value (V)	98.50	98.25	98.23	98.25	98.27
Present Value of payments (A)	107.87	107.74	107.73	107.74	107.65
Prepayment Option Value (C)	9.09	9.36	9.49	9.31	9.21
Default Option Value (D)	0.28	0.13	0.00	0.18	0.18
Panel B: 90% LTV					
Points	1.50	2.21	1.97	1.79	1.74
Mortgage Value (V)	98.50	97.79	98.03	98.21	98.26
Present Value of payments (A)	109.96	112.61	109.36	109.10	109.09
Prepayment Option Value (C)	9.66	14.82	11.32	9.78	9.60
Default Option Value (D)	1.80	0.00	0.00	1.11	1.24
Panel C: 95% LTV					
Points	1.50	1.93	2.21	1.79	1.75
Mortgage Value (V)	98.50	98.07	97.79	98.21	98.25
Present Value of payments (A)	113.72	112.65	112.61	112.03	112.21
Prepayment Option Value (C)	10.68	12.13	14.82	10.76	10.55
Default Option Value (D)	4.54	2.44	0.00	3.07	3.41

Note: This table reports the numerical analysis holding the interest rate constant at the standard FRM contract rate and varying the total points paid at origination. The analysis of the FRM and variations of the ABM is reported for three common loan-to-value (LTV) ratio assumptions. The contract rates at origination are 5.73%, 5.90%, and 6.22% for the 80%, 90%, and 95% LTV mortgages, respectively. The table reports the equilibrium origination values for the present value of the mortgage payments (A), the value of the borrower's prepayment option (C), and the value of the default option (D).

Table 4: **Impact of Interest Rate and House Price Volatility on the FRM and ABM Contracts**

	Standard Model				Firstloss Model - Annual				Firstloss Model - Quarterly			
	$\sigma_H = 5\%$ $\sigma_r = 5\%$	$\sigma_H = 5\%$ $\sigma_r = 15\%$	$\sigma_H = 15\%$ $\sigma_r = 5\%$	$\sigma_H = 15\%$ $\sigma_r = 15\%$	$\sigma_H = 5\%$ $\sigma_r = 5\%$	$\sigma_H = 5\%$ $\sigma_r = 15\%$	$\sigma_H = 15\%$ $\sigma_r = 5\%$	$\sigma_H = 15\%$ $\sigma_r = 15\%$	$\sigma_H = 5\%$ $\sigma_r = 5\%$	$\sigma_H = 5\%$ $\sigma_r = 15\%$	$\sigma_H = 15\%$ $\sigma_r = 5\%$	$\sigma_H = 15\%$ $\sigma_r = 15\%$
Panel A:												
LTV = 80%												
Contract Rate	5.116%	5.811%	5.626%	6.616%	5.177%	6.001%	5.811%	6.907%	5.177%	6.001%	5.856%	8.130%
Mortgage Value (V)	98.50	98.50	98.50	98.50	98.50	98.50	98.50	98.50	98.50	98.50	98.50	98.50
Present Value of payments (A)	101.20	116.46	103.71	120.77	101.57	118.44	104.25	121.49	101.57	118.44	104.73	134.59
Prepayment Option Value (C)	2.70	17.96	3.04	19.97	3.07	19.94	5.58	22.29	3.07	19.94	6.23	36.09
Default Option Value (D)	0.00	0.00	2.17	2.30	0.00	0.00	0.17	0.71	0.00	0.00	0.00	0.00
Panel B:												
LTV = 90%												
Contract Rate	5.118%	5.814%	7.108%	7.315%	5.179%	6.004%	6.645%	7.795%	5.180%	6.004%	7.036%	8.134%
Mortgage Value (V)	98.50	98.50	98.50	98.50	98.50	98.50	98.50	98.50	98.50	98.50	98.50	98.50
Present Value of payments (A)	101.22	116.50	121.27	129.68	101.59	118.44	112.55	130.56	101.60	118.44	116.97	134.64
Prepayment Option Value (C)	2.70	17.95	12.74	23.20	3.08	19.91	12.92	28.88	3.10	19.92	18.47	36.14
Default Option Value (D)	0.02	0.06	10.03	7.98	0.01	0.03	1.13	3.18	0.01	0.02	0.00	0.00
Panel C:												
LTV = 95%												
Contract Rate	5.134%	5.852%	7.108%	8.265%	5.198%	6.041%	7.956%	8.927%	5.201%	6.041%	9.495%	9.960%
Mortgage Value (V)	98.50	98.50	98.50	98.50	98.50	98.50	98.50	98.50	98.50	98.50	98.50	98.50
Present Value of payments (A)	101.40	116.99	121.27	142.18	101.79	118.82	126.88	143.28	101.82	118.82	145.32	156.14
Prepayment Option Value (C)	2.72	17.73	12.74	27.79	3.15	19.75	22.11	34.43	3.26	19.75	46.82	57.64
Default Option Value (D)	0.19	0.75	10.03	15.88	0.14	0.57	6.27	10.35	0.06	0.57	0.00	0.00

Note: This table reports the numerical analysis assuming differences in interest rate volatility (σ_r) and house price volatility (σ_H). The analysis of the FRM and variations of the ABM is reported for three common loan-to-value (LTV) ratio assumptions. The mortgages are priced in equilibrium assuming a par value of $98.50. The table reports the equilibrium origination values for the present value of the mortgage payments (A), the value of the borrower's prepayment option (C), and the value of the default option (D).
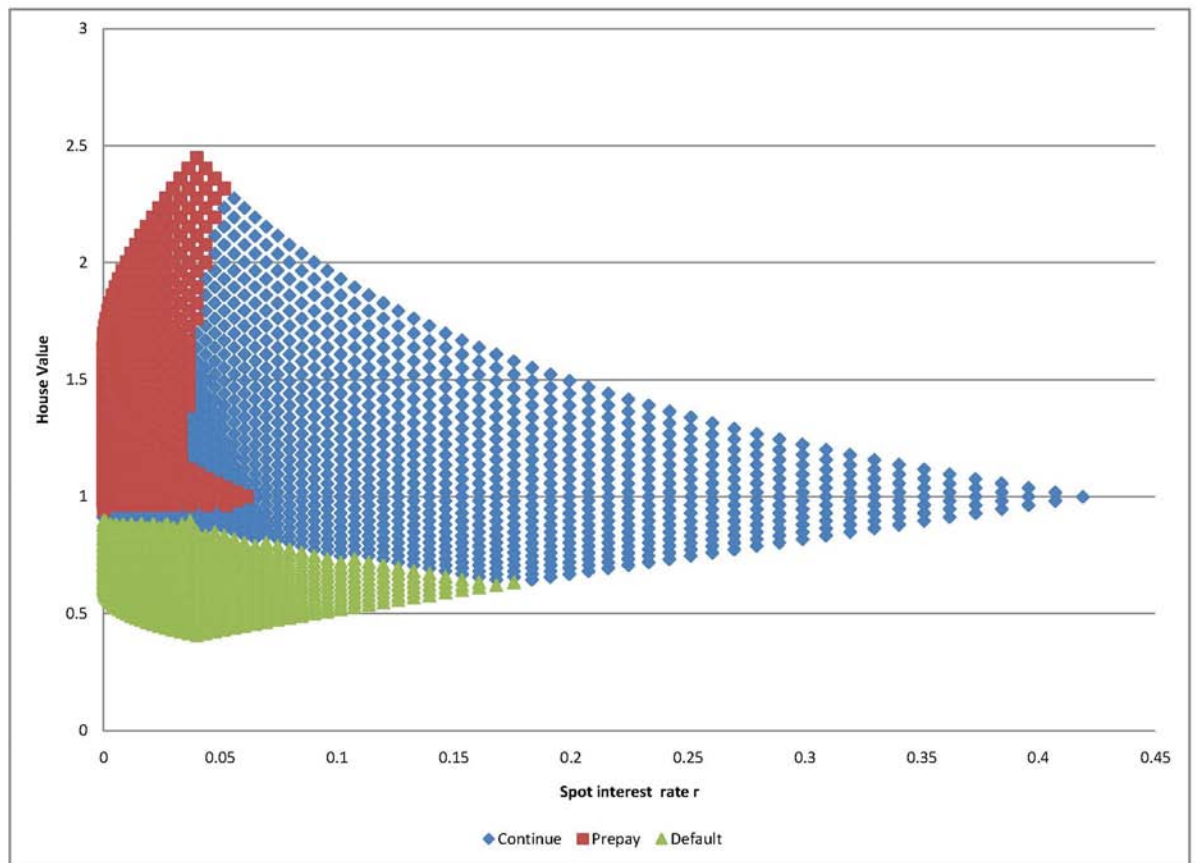
27

Figure 1: **Borrower Actions at Month 36 for the Standard Fixed-Rate Mortgage (FRM)**
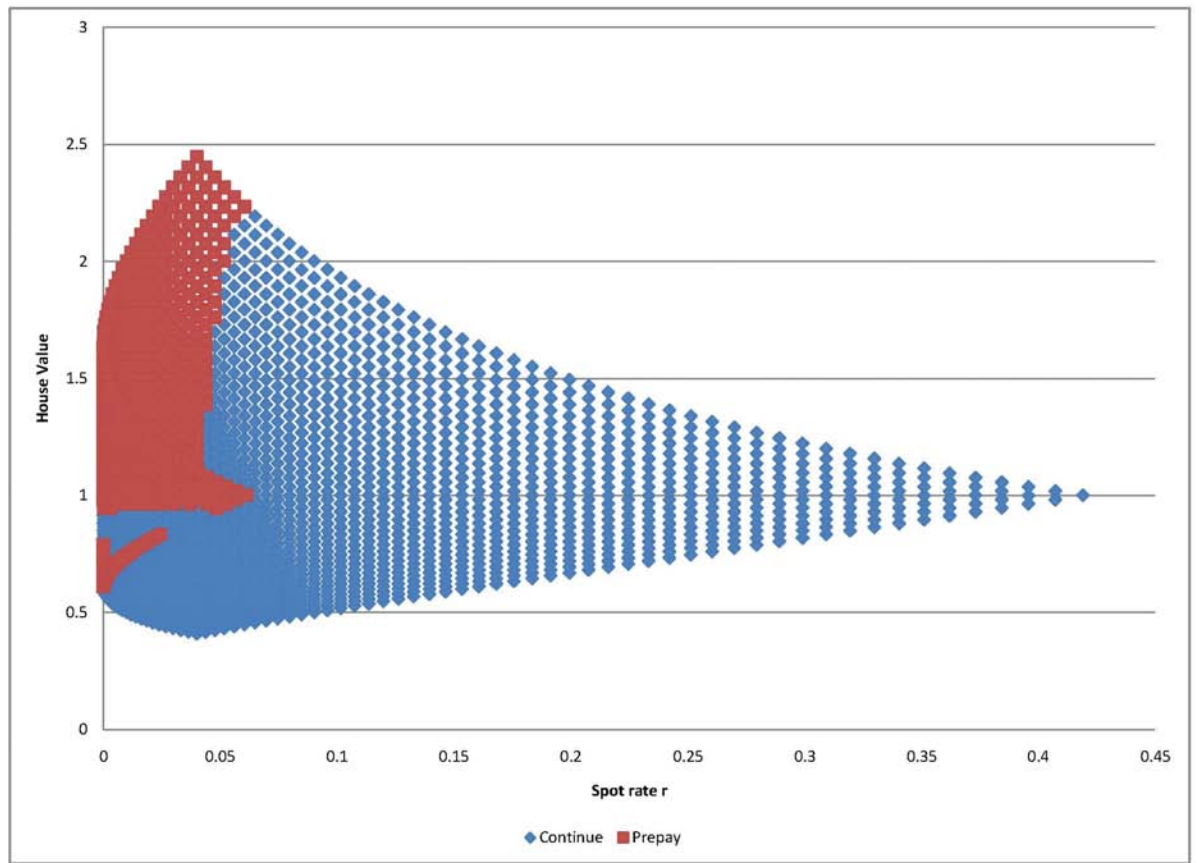
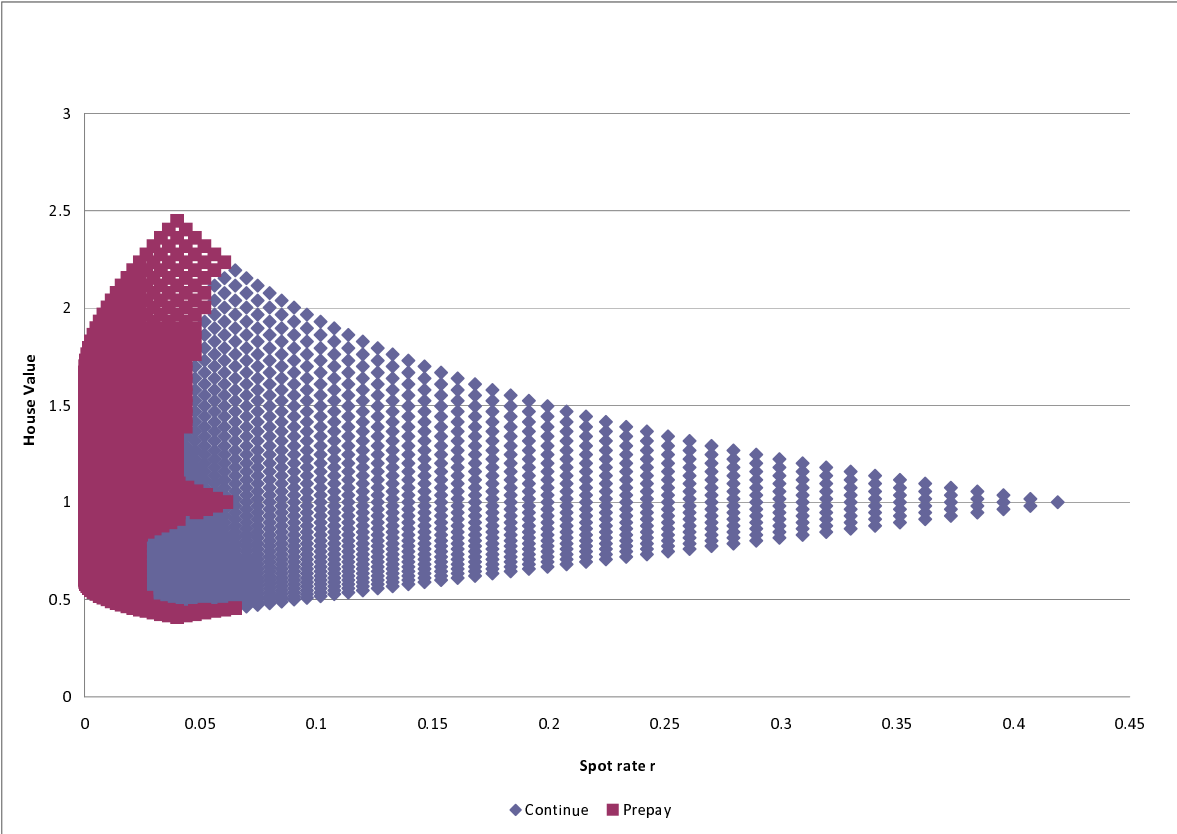Figure 2: **Borrower Actions at Month 36 for the Quarterly ABM**

Figure 3: **Borrower Actions at Month 36 - Annual Adjustable Balance Mortgage**
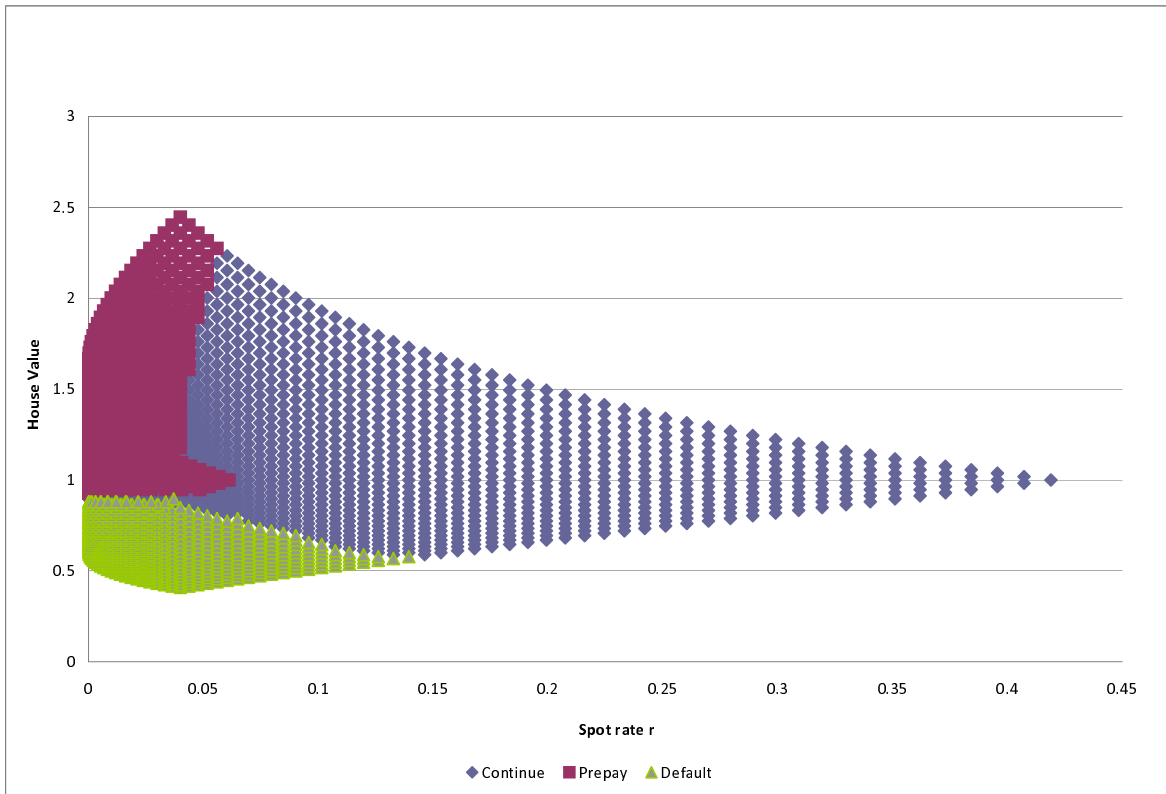Conditional upon mortgage still being in force.

Figure 4: **Borrower Actions at Month 42 - Annual Adjustable Balance Mortgage**
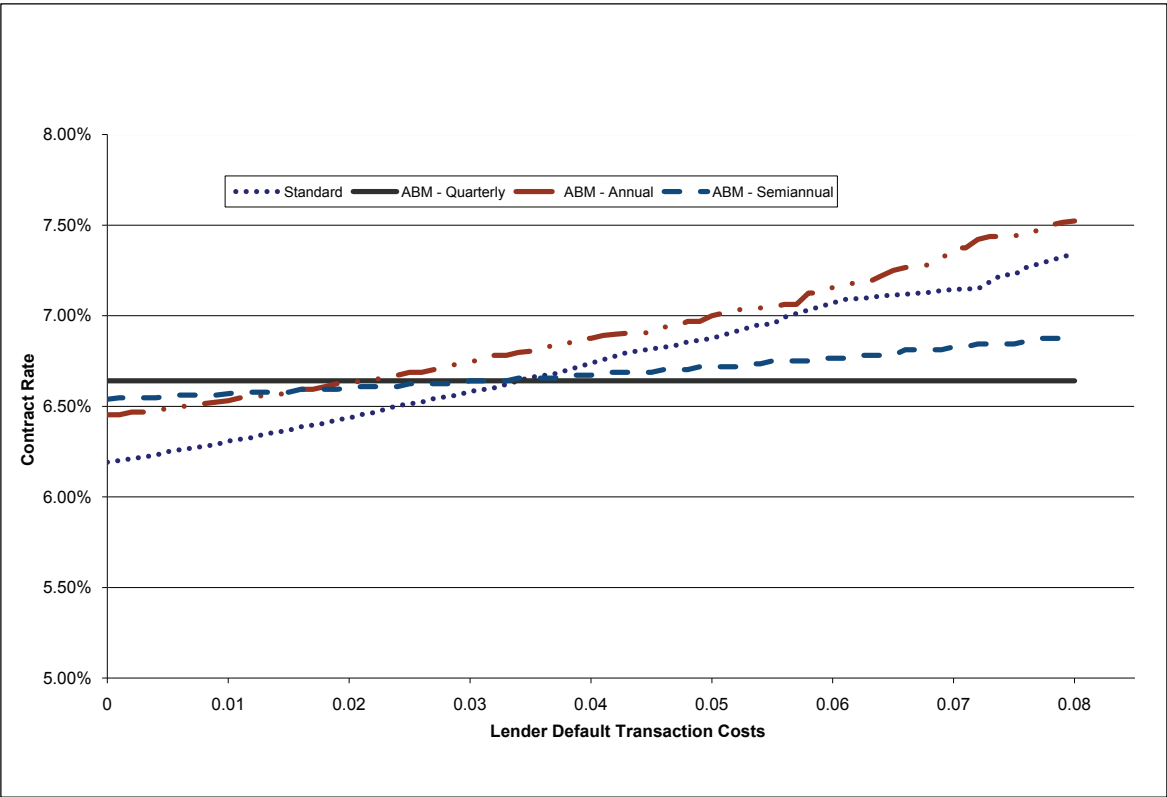Conditional upon mortgage still being in force.

Figure 5: **Equilibrium Contract Rates By Mortgage Type Given Various Lender Default Transaction Costs**
95% LTV Case